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GEOMETRY.

112. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio State University, Athens, Ohio.

The tangent planes at A, B, C, D to the sphere circumscribing the tetrahedron $ABCD$ form a tetrahedron $abcd$; prove that Aa, Bb, Cc, Dd will meet in a point if $BC \cdot AD = CA \cdot BD = AB \cdot CD$.

I. Solution by the PROPOSER.

Using tetrahedral coördinates, the equation to the sphere circumscribing the tetrahedron of reference is

$$2a^2\beta\gamma + 2b^2\alpha\gamma + 2c^2\alpha\beta + 2a'^2\alpha\delta + 2b'^2\beta\delta + 2c'^2\gamma\delta = 0 \dots\dots (1),$$

a, b, c being the sides of the base of the tetrahedron, and a', b', c' the edges opposite.

The polar plane of A (1, 0, 0, 0) is $c^2\beta + b^2\gamma + a'^2\delta = 0 \dots\dots (2),$

of $B,$ $c^2\alpha + a^2\gamma + b'^2\delta = 0 \dots\dots (3),$

of $C,$ $b^2\alpha + a^2\beta + c'^2\delta = 0 \dots\dots (4),$

and of $D,$ $a'^2\alpha + b'^2\beta + c'^2\gamma = 0 \dots\dots (5).$

Now assuming, for simplicity, $aa' = bb' = cc' \dots\dots (6),$ and calling the vertices of the tetrahedron opposite (2), (3), (4), (5), $a, b, c, d,$ we have for the coördinates of $a,$

$$\alpha_1 = \frac{2a^4a'^2}{a^2b^2c^2 - 2a^4c'^2 + a^2c^2a'^2 + a^2b^2a'^2} = -\frac{2a^4a'^2}{D_1} \dots\dots (7).$$

$$\beta_1 = \frac{b^4b'^2}{D_1} \dots\dots (8), \quad \gamma_1 = \frac{c^4c'^2}{D_1} \dots\dots (9), \quad \delta_1 = \frac{a^2b^2c^2}{D_1} \dots\dots (10).$$

Then the equation to the line Aa is

$$\frac{1-\alpha}{a^2b^2c^2 + b^2a^2a'^2 + c^2a^2a'^2} = \frac{\beta}{a^2b^2a'^2} = \frac{\gamma}{a^2c^2a'^2} = \frac{\delta}{a^2b^2c^2} \dots\dots (11).$$

Similarly, the equations to $Bb, Cc, Dd,$ are, respectively,

$$\frac{\alpha}{a^2b^2b'^2} = \frac{1-\beta}{a^2b^2c^2 + a^2b^2b'^2 + b^2c^2b'^2} = \frac{\gamma}{b^2c^2b'^2} = \frac{\delta}{a^2b^2c^2} \dots\dots (12).$$

$$\frac{\alpha}{a^2c^2c'^2} = \frac{\beta}{b^2c^2c'^2} = \frac{1-\gamma}{a^2b^2c^2 + b^2c^2c'^2 + a^2c^2c'^2} = \frac{\delta}{a^2b^2c^2} \dots\dots (13),$$

$$\frac{\alpha}{a^2b^2b'^2} = \frac{\beta}{b^2c^2c'^2} = \frac{\gamma}{a^2c^2a'^2} = \frac{1-\delta}{a^2b^2b'^2 + b^2c^2c'^2 + a^2c^2a'^2} \dots\dots (14).$$

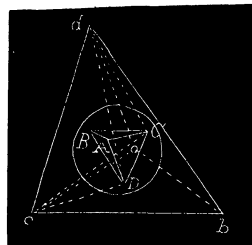
(11), (12), (13), (14) are concurrent.

II. Solution by B. F. YANNEY, A. M., Professor of Mathematics in Mount Union College, Alliance, O.

Let $ABCD$ be the tetrahedron about which is circumscribed the sphere; and $abcd$ the tetrahedron formed by planes tangent respectively at A, B, C , and D . Given, also, that $BC \cdot AD = CA \cdot BD = AB \cdot CD$.

To prove that Aa, Bb, Cc, Dd meet in a point, as at O .

The points c, D, C , and d are in the same plane. For D and C have their respective distances from the planes cbd and cad in the same ratio, viz: AD^2/AC^2 and BD^2/BC^2 , which are equal, since by hypothesis $BC \cdot AD = AC \cdot BD$.



That first part of preceding statement is true is evident from the fact that if diameters are supposed to be drawn from A and B respectively, the respective projections upon these diameters of the chords AD, AC , and BD, BC , have the same ratios respectively as the squares of the chords; and the projections equal the distances respectively from the points D and C to the planes.

Similarly, co-planar are the points c, B, C, b ; b, A, B, a ; and a, D, A, d . Therefore, Dd and Cc intersect; so, also, Dd and Bb , and Cc and Bb .

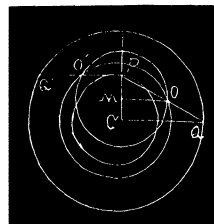
113. Proposed by T. W. PALMER, Professor of Mathematics, University of Alabama.

Given three concentric circles. Draw a straight line from the inner to the outer circumference that shall be bisected by the middle circumference.

I. Solution by C. HORNING, A. M., Professor of Mathematics, Heidelberg University, Tiffin, O., and J. C. GREGG, A. M., Superintendent of Schools, Brazil, Ind.

Let C be the common center and CP the radius of the inner circle, and CQ that of the outer circle.

Bisect CP at M and with one-half of CQ as radius and M as center describe a circle cutting the middle circumference at O or O' . Draw PO and produce to the outer circumference at Q . Then POQ is the required line. For, $\because PM = MC$ and $MO = \frac{1}{2}CQ$, $\therefore PO = OQ$.



According as $\frac{1}{2}$ radius of outer circle is greater than, equal to, or less than the radius of the middle circle increased by $\frac{1}{2}$ radius of inner circle, there are two solutions, one solution, or no solution.

II. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.; J. SCHEFFER, A. M., Hagerstown, Md.; COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; J. C. NAGLE, C. E., Professor of Civil Engineering, Agricultural and Mechanical College of Texas, College Station, Tex.; CHAS. C. CROSS, Libertytown, Md.; J. O. MAHONEY, B. E., M. Sc., Teacher of Mathematics and Science in Carthage High School, Carthage, Tex., and the PROPOSER.

Let O be the center of the circles, and A any point on the inner circumference. With O as a center and a radius = 2 times the radius of the middle circle, and with A as a center and a radius = the radius of the outer circle, describe two arcs intersecting at D . Draw OD intersecting middle circumference at B .